

GREY KANGAROO Thursday 16 March 2023

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a member of the Association Kangourou sans Frontières

England & Wales: Year 9 or below Scotland: S2 or below Northern Ireland: Year 10 or below

INSTRUCTIONS

- 1. Do not open the paper until the invigilator tells you to do so.
- 2. Time allowed: **60 minutes**. No answers, or personal details, may be entered after the allowed time is over.
- 3. The use of blank or lined paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
- 4. Use a B or an HB non-propelling pencil. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
- 5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
- 6. Scoring rules:

5 marks are awarded for each correct answer to Questions 1-15; 6 marks are awarded for each correct answer to Questions 16-25; In this paper you will not lose marks for getting answers wrong.

- 7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way, or reject the answer sheet.
- 8. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Grey Kangaroo should be sent to:

challenges@ukmt.org.uk www.ukmt.org.uk 1. Which of the shapes below cannot be divided into two trapeziums by a single straight line? В С E A D triangle rectangle trapezium regular square hexagon 2. What is the sum of the largest three-digit multiple of 4 and the smallest four-digit multiple of 3? A 1996 B 1997 C 1998 D 1999 E 2000 3. Werner wants to write a number at each vertex and on each edge of the 8 9 rhombus shown. He wants the sum of the numbers at the two vertices at the ends of each edge to be equal to the number written on that edge. What number should he write on the edge marked with the question mark? 13 C 13 A 11 B 12 D 14 E 15 4. Kristina has a piece of transparent paper with some lines marked on it. She folds it along the central dashed line, as indicated. What can she now see? B 2:6:6 C 5:6:9 D 5:8:2 E А 2:6:9 5:8:9 5. John has 150 coins. When he throws them on the table, 40% of them show heads and 60% of them show tails. How many coins showing tails does he need to turn over to have the same number showing heads as showing tails? A 10 C 20 D 25 E 30 B 15 6. Anna has five circular discs, each of a different size. She decides to build a tower using three of her discs so that each disc in her tower is smaller than the disc below it. How many different towers could Anna construct? A 5 B 6 C 8 D 10 E 15 7. Evita wants to write the numbers 1 to 8 in the boxes of the grid shown, so that 4 the sums of the numbers in the boxes in each row are equal and the sums of the 3 8 numbers in the boxes in each column are equal. She has already written numbers 3, 4 and 8, as shown. What number should she write in the shaded box? A 1 **B** 2 C 5 D 6 E 7 8. Theodorika wrote down four consecutive positive integers in order. She used symbols instead of digits. She wrote the first three integers as $\Box \diamondsuit \diamondsuit$, $\heartsuit \triangle \triangle$, $\heartsuit \triangle \Box$. What would she write in place of the next integer in the sequence? $A \heartsuit \heartsuit \diamondsuit$ $B \square \heartsuit \square$ $C \heartsuit \triangle \diamondsuit$ $D \heartsuit \diamondsuit \Box$ $E \heartsuit \triangle \heartsuit$ 9. The diagram shows five equal semicircles and the lengths of some line segments. 12 12 What is the radius of the semicircles? 22 22 16 A 12 B 16 C 18 D 22 E 28

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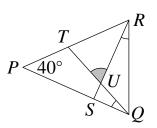
E 6

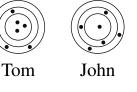
0123456789

- **10.** Some edges of a cube are to be coloured red so that every face of the cube has at least one red edge. What is the smallest possible number of edges that could be coloured red?
 - A 2 B 3 C 4 D 5
- **11.** Matchsticks can be used to write digits, as shown in the diagram. How many different positive integers can be written using exactly six matchsticks in this way?
 - A 2 B 4 C 6 D 8 E 9
- **12.** A square with side-length 10 cm long is drawn on a piece of paper. How many points on the paper are exactly 10 cm away from two of the vertices of this square?
 - A 4 B 6 C 8 D 10 E 12
- **13.** In the diagram shown, sides *PQ* and *PR* are equal. Also $\angle QPR = 40^{\circ}$ and $\angle TQP = \angle SRQ$. What is the size of $\angle TUR$?
 - A 55° B 60° C 65° D 70° E 75°
- 14. Tom, John and Lily each shot six arrows at a target. Arrows hitting anywhere within the same ring scored the same number of points. Tom scored 46 points and John scored 34 points, as shown. How many points did Lily score?
 - A 37 B 38 C 39 D 40 E 41
- 15. The diagram shows a smaller rectangle made from three squares, each of area 25 cm², inside a larger rectangle. Two of the vertices of the smaller rectangle lie on the mid-points of the shorter sides of the larger rectangle. The other two vertices of the smaller rectangle lie on the other two sides of the larger rectangle. What is the area, in cm², of the larger rectangle?
 - A 125 B 136 C 149 D 150 E 172
- **16.** The sum of 2023 consecutive integers is 2023. What is the sum of digits of the largest of these integers?

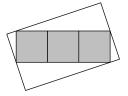
A 4 B 5 C 6 D 7 E 8

- **17.** Some beavers and some kangaroos are standing in a circle. There are three beavers in total and no beaver is standing next to another beaver. Exactly three kangaroos stand next to another kangaroo. What is the number of kangaroos in the circle?
 - A 4 B 5 C 6 D 7 E 8
- 18. Snow White organised a chess competition for the seven dwarves, in which each dwarf played one game with every other dwarf. On Monday, Grumpy played 1 game, Sneezy played 2, Sleepy 3, Bashful 4, Happy 5 and Doc played 6 games. How many games did Dopey play on Monday?
 - A 1 B 2 C 3 D 4 E 5









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19. Elizabetta wants to write the integers 1 to 9 in the regions of the shape shown, with one integer in each region. She wants the product of the integers in any two regions that have a common edge to be not more than 15. In how many ways can she do this?

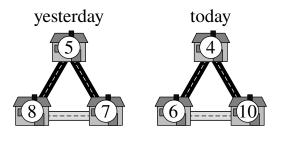
A 8 B 12 C 16 D 24 E 32

20. There were twice as many children as adults sitting round a table. The age of each person at the table was a positive integer greater than 1. The sum of the ages of the adults was 156. The mean age of the children was 80% less than the mean age of the whole group. What the sum of the ages of the children?

- A 10 B 12 C 18 D 24 E 27
- **21.** Martin is standing in a queue. The number of people in the queue is a multiple of 3. He notices that he has as many people in front of him as behind him. He sees two friends, both standing behind him in the queue, one in 19th place and the other in 28th place. In which position in the queue is Martin?

A 14 B 15 C 16 D 17 E 18

22. Some mice live in three neighbouring houses. Last night, every mouse left its house and moved to one of the other two houses, always taking the shortest route. The numbers in the diagram show the number of mice per house, yesterday and today. How many mice used the path at the bottom of the diagram ?



E 6

1015

A 9 B 11 C 12 D 16 E 19

D 5

A 2 B 3 C 4

24. Jake wrote six consecutive numbers on six white pieces of paper, one number on each piece. He stuck these bits of paper onto the top and bottom of three coins. Then he tossed these three coins three times. On the first toss, he saw the numbers 6, 7 and 8 and then coloured them red. On the second toss, the sum of the numbers he saw was 23 and on the third toss the sum was 17. What was the sum of the numbers on the remaining three white pieces of paper?

A 18 B 19 C 23 D 24 E 30

25. A rugby team scored 24 points, 17 points and 25 points in the seventh, eighth and ninth games of their season. Their mean points-per-game was higher after 9 games than it was after their first 6 games. What is the smallest number of points that they could score in their 10th game for their mean number of points-per-game to exceed 22?

A 22 B 23 C 24 D 25 E 26